Kiora Hall

Calculation Policy



INTRODUCTION

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in stage blocks under the following headings: addition, subtraction, multiplication and division.
Statements taken directly from the programmes of study are listed in bold at the beginning of each section.

A separate mental maths policy outlines mental calculation strategies, including the use of jottings, vocabulary to be developed and the key number facts that learners will need to know.

Learners will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that learners develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that learners can use these methods accurately with confidence and understanding

HOW TO USE THIS POLICY

- Use the policy as the basis of your planning but ensure you use previous or following Stages' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of learners
- If, at any time, learners are making significant errors, return to the previous stage in calculation
- Always use suitable resources, models and images to support learners's understanding of calculation and place value, as appropriate
- Encourage learners to make sensible choices about the methods they use when solving problem

Baseline Assessment

On entry to Kiora Hall, each learners undergoes a 6-week extended initial assessment (EIA). Throughout this assessment focus we will initially be focused on how to remove barriers to learning for the learners, ensuring they can access a purposeful curriculum. The EIA are particularly important due to the diverse learning profiles of pupils. Implementing these assessments effectively can involve:

- **Utilising a Range of Tools:** Employ various assessment tools to capture a comprehensive picture of each pupil's abilities in areas such as communication, social skills, and academic knowledge.
- **Collaborative Approach:** All staff supporting to assess the best approach to ensure learners needs are met.
- **Continuous Review:** Regularly review and update baseline data to reflect changes in learners' needs and adjust instructional strategies accordingly.

By prioritising baseline assessments, we can ensure that all pupils receive the personalised support they need to thrive.

Details of the baseline assessment are then used to develop the learners's STEP plan. The next element of the assessment is to gauge the learners's current academic attainment and speed of learning, this aids the teaching team in planning how we can support the learners to reach their planned destination, information on the baseline assessment and planned destination is held within the learners's individual education plan (IEP). Once an accurate assessment of the learners is formed, which will indicate the current stage of learning, gaps in learning and misconceptions; teachers will plan a bespoke curriculum for the learners. Each learners will begin their own learning pathway. Learners will begin on either sensory and communication pathway or education pathway. Those learners on the sensory and communication pathway would normally follow milestones whereas learners on an education pathway tend to follow stages. At a later stage in the learners pathway, there will be a focus on functional skills development.

Small Steps Approach

Throughout each stage of learning, learners will be taught through progressive blocks of learning that are made up of small steps. These small steps are coherent and connected to ensure all learners, no matter of their ability, can access the lesson. The small steps approach ensures learners have sufficient opportunities to embed mathematical concepts in a range of contexts. To support in embedding concepts in a purposeful manner, maths is contextualised within the preparing for adulthood curriculum (PfA). This curriculum ensures that learners can put their skills in to practice in everyday situations that is applicable to them living as independent life as possible.

Working in small steps allows learners to develop a mastery knowledge of the subject, as concepts are broken down to a level that a learners can master without feeling over-whelmed. In addition to schemes being small steps, each lesson is broken down into smaller chunks, ensuring that every learners finds each lesson accessible and through personalised curriculum programmes and high levels of environmental and staff support, learners can supported to keep up rather than needing to catch up.

A key element of the small steps approach is a learners does not move on to the next step until they have been fully exposed to the step that they are currently working towards. We are aware that all learners at Mackenzie Thorpe Centre progress at different rates and will need varying amounts of time working at each level and due to the nature of the personalised curriculum we are able to meet the needs of all learners.

Progression across a learners's school life is visible within the school's progression map, this is then divided in to clear blocks of learning that are shown on the long term topic maps for each class and within teachers' medium term planning. Teachers' short-term planning indicates how learning will be contextualised and where links are made to the PfA curriculum.

Retrieval Practice

To ensure learners are making connections between the sequential chunks of learning, teachers will make regular use of retrieval practice at the start of every lesson and make an explicit link as to how this will link with their new learning. The use of retrieval at the start of every session will also ensure that learners are ready for the next step in the sequence of learning.

We encourage a range of retrieval practice strategies in our lessons:

- **Daily or Weekly Quizzes:** Incorporate short quizzes at the start of each lesson to review past topics. These quizzes can be low-stakes to reduce anxiety and encourage participation.
- **Flashcards:** Use flashcards for key concepts and formulas, allowing pupils to test themselves and each other in pairs or small groups.
- **Mind Maps:** Encourage pupils to create mind maps of what they've learned in previous lessons, connecting new and old information visually.
- **Peer Teaching:** Have pupils explain concepts to each other, promoting retrieval through teaching and discussion.
- **Interactive Activities:** Use games or interactive activities that require recalling and applying past knowledge in fun and engaging ways.

CPA Approach

The concrete, pictorial and abstract (CPA) approach is used for the teaching of all mathematical concepts at Mackenzie Thorpe Centre. All elements are to be used with the whole class and the teacher will promote each element of the approach equally. Concrete manipulatives are never used as a tool just to support lower achieving learners; they support concepts for all learners.

<u>Concrete</u>

'…abstract notions of mathematics may only be accessible to them through embodiment in practical resources…' (Piaget, 1951). It is a key principle of the maths strategy at Mackenzie Thorpe Centre that all learners experience mathematical concepts using concrete manipulatives. This principle applies to all ages and abilities as to develop any deep understanding of mathematical concepts learners must be able to visually see the mathematical patterns. The purpose of the concrete stage within the CPA process is to allow the learners to investigate a concept and make connections to previously learnt skills.

Concrete Resources – Available in all classrooms
Base 10
Number Lines
Numicon
Double Sided Counters
Dice
Blocks
Place Value Charts
Lolly Pop Sticks
Pattern Blocks
Geo Boards

<u>Pictorial</u>

The purpose of the pictorial stage is to sustain understanding of mathematical concepts and processes. This stage provides learners with the skills to represent maths questions in a way which allows them to clarify the question.

<u>Abstract (Contextualised Learning Opportunities)</u>

The abstract stage is not a third step in the process, but rather woven within the concrete and pictorial stage. This stage allows our learners to forge meaningful links between what they are learning in the classroom and applying the concept in practical situations, that link to their planned destination. This supports the learners at Mackenzie Thorpe Centre in working towards living as independent life as possible, taking in to account their own support needs.

'If we do not use concrete manipulations, then we cannot understand mathematics. If we only use concrete manipulations, we are not doing mathematics.' (Gu, 2015)

Contextualised Learning Opportunities

To ensure learning is purposeful for all learners, every opportunity is taken at Kiora Hall to contextualise learning. This begins in the classroom, by forming real life links between the skills learners are developing and how they can be applied outside of the classroom. This learning is then extended by practising the skills within the local community through a vast range of activities. Through our post-14 curriculum, learners also can access work experience and simulated work experience, allowing them to understand how the skills they have developed in the classroom can be applied in the workplace. For those learners, where work experience is not suitable, they will be supported in understanding how the skills they have developed apply to independent living. Some examples, including skills and teaching methods are:

Money Management:

- **Skills:** Counting money, making change, budgeting, and understanding bank statements.
- **Teaching Methods:** Use role-playing scenarios such as shopping exercises, where pupils practice using real or play money to buy items. Introduce budgeting activities where pupils plan and manage a simple budget.

Time Management:

- **Skills:** Reading clocks, understanding time duration, and scheduling.
- **Teaching Methods:** Use analog and digital clocks to teach time-telling. Encourage pupils to create daily schedules and use timers or alarms to manage tasks and transitions.

Measurement:

- **Skills:** Understanding and using units of measurement (length, weight, volume) in cooking, shopping, and DIY tasks.
- **Teaching Methods:** Engage pupils in cooking classes where they measure ingredients. Use practical tasks like measuring items for a project or comparing weights.

Numeracy and Problem Solving:

- **Skills:** Basic arithmetic operations, understanding proportions, and solving real-life problems.
- **Teaching Methods:** Present real-world problems such as planning a trip or organizing an event that requires calculations and decision-making.

Understanding Bills and Expenses:

- **Skills:** Reading and interpreting utility bills, understanding expenses, and managing subscriptions.
- **Teaching Methods:** Use sample bills to teach pupils how to read and understand them. Discuss common household expenses and how to track them.

Using Technology:

- **Skills:** Using calculators, apps, and online banking tools to manage finances.
- **Teaching Methods:** Introduce pupils to calculators and budgeting apps. Conduct workshops on how to use online banking safely.

Engagement Ideas

To support our learners at Mackenzie Thorpe, we aim to use a range of strategies to engage the learners and focus on their strengths. Below is an example of ways in which we engage learners to progress and achieve to their full potential.

Visual Schedules and Timers:
Use visual schedules to outline the steps of a math problem. For example, breaking down a long division problem into a sequence of steps with visual cues can help learners follow along more easily
Timers can be used to allocate specific time slots for each step, helping students manage their time and stay focused
Visual schedules and timers will be used, where appropriate. Alternatively, learners will be given the time needed to complete work. This is learners dependent and suited to their needs.

Manipulatives and Hands-On Activities:
Use physical objects like counting blocks, beads, or abacuses to represent numbers and operations. For instance, using blocks to demonstrate addition and subtraction can make these concepts more tangible
Interactive activities, such as using a number line on the floor where students can physically step to the correct answer, can also be very engaging

Incorporating Interests:
If a learners has a particular interest, such as trains or animals, incorporate these themes into math problems. For example, if a learners loves trains, you could create a problem that involves calculating the distance between train stations

Technology and Apps:
Utilise educational apps and software that offer interactive math games and exercises.
Virtual manipulatives, such as online abacuses or interactive number lines, can also be very helpful

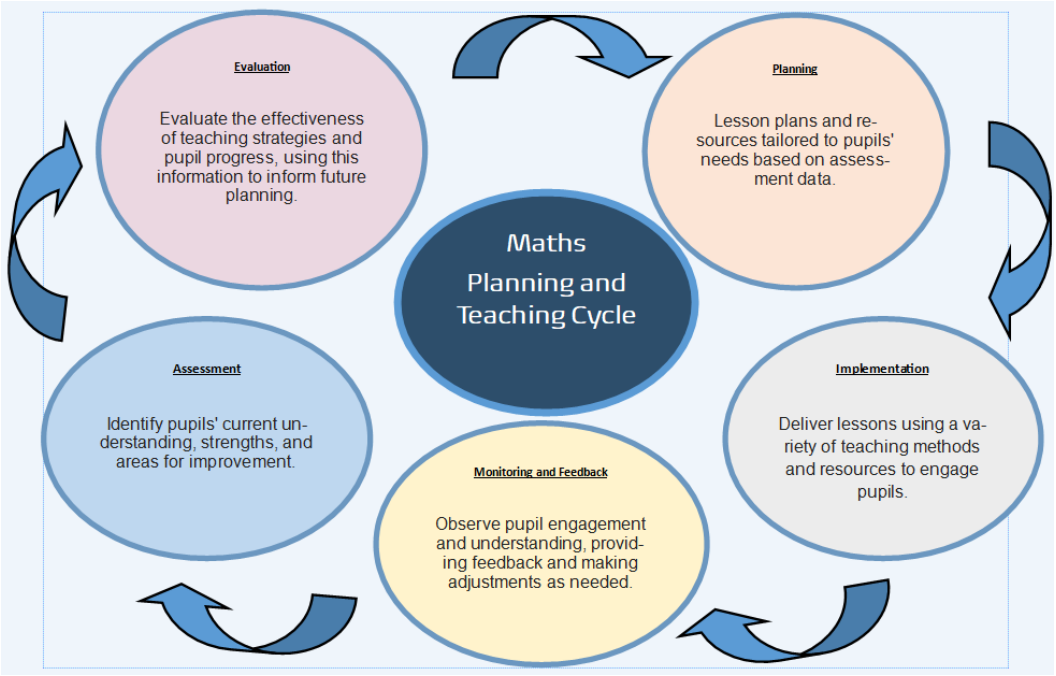
Step-by-Step Instructions:
Provide clear, step-by-step written instructions for solving math problems. For example, a worksheet might include a detailed breakdown of how to solve a multiplication problem, with each step clearly labelled
Use color-coding to highlight different parts of the problem or different steps in the process

Multi-Sensory Techniques:
Engage multiple senses by combining visual, auditory, and tactile elements. For example, students can use sandpaper numbers to trace while saying the number out loud, reinforcing the concept through touch, sight, and sound

Fluency, Problem Solving and Reasoning

All learners, no matter of ability, should be exposed to all three elements of the approach. Each element should be delivered in a manner whereby they run alongside each other. It is also imperative that the CPA approach is used for the delivery of all three aspects of learning and not used exclusively for fluency based practice.

To aid the learners understanding and to support in forming meaningful links between the three areas of mathematical thinking, Mackenzie Thorpe Centre follows their own maths and planning cycle to support the planning and teaching of maths.



Fluency

We have a strong emphasis on fluency at Mackenzie Thorpe Centre, fluency in mathematics is a broad concept and is the bedrock of effective teaching and learning. It supports learners to develop an understanding of all mathematical concepts using concrete, pictorial and abstract approaches. Being fluent within a mathematical concept avoids the learners becoming cognitively overloaded when dealing with problem solving and reasoning situations.

To ensure learners become fluent in mathematical concepts, all learners will be exposed to daily fluency activities that link to prior learning. The level and structure of these activities will be dependent upon the needs of individual learners.

Problem Solving/ Contextualised Learning

Ensuring competency in problem solving and ability to apply skills in everyday life, with the aim of developing independent problem solvers is a key aim of the maths curriculum at Mackenzie Thorpe Centre. In developing problem-solving skills and strategies learners will be encouraged to;

- Apply the concepts and skills they have learnt,
- Use and compare different mathematical approaches

- Independently break down problems into a series of simpler steps
- Persevere in seeking solutions
- Reflect on, and communicate their problem-solving ideas and strategies

Resources and practical equipment should be used at every opportunity to ensure that learners have a greater depth understanding of the problem-solving process. A conscious effort will always be made to link problem solving with real issues that the learners will face, supporting the preparing for adulthood curriculum.

Reasoning

Reasoning activities provide learners with the opportunity to apply their learnt skills and conceptual understanding in a variety of different concepts. Reasoning is the process of learners logically applying their critical thinking to work out the best strategy or process to follow. The reasoning stage can be seen as the stage where the glue bonds together the learner's mathematical skills; it also allows learners to bridge the gap between fluency and problem solving.

Specialist Strategies

The use of specialist strategies to ensure we remove barriers to learning and to allow all learners to succeed is the golden thread of our curriculum at Mackenzie Thorpe Centre. As with all subjects, staff will support the learners to check the correct communication aids are available (symbols, PECS, objects of reference, concrete resources and visual representations) so that all learners are able to access the curriculum at a stage appropriate level. We also ensure that learners receive high levels of support with staffing ratios, but through the effective use of hierarchy of cues, learners can develop a good level of independence.

Formal Curriculum

In the Formal Curriculum specific maths lessons will be seen, as learners start to understand and apply concepts independently, for example working out how much a shopping list will cost. They will also begin to be able to work abstractly and use common mathematical language such as divide, multiply or addition. There is a need to be assured that young people have deeply learnt concepts, i.e. they can apply concepts in a functional way or to new situations. Care is taken to avoid the trap of setting targets as means of progressing understanding as opposed to check that learnt concepts can be used across a variety of situations. This is of important to learners such as those who may be good at spotting patterns but less able to understand why the pattern exists. For older learners, formal qualifications will be considered as pathway into future college choices or employment, for these learners part of the considerations will need to be both the level of qualification and the exam board used to avoid the risk that the curriculum becomes about passing an exam and not about extending mathematical understanding and its application. Kiora Hall use WhiteRose maths to provide support and structure with the maths curriculum.

Post-16 Curriculum

The Post-16 Curriculum is set up to enable young people to move beyond Mackenzie Thorpe Centre, this is of course a very individual step as such their curriculum reflects and teaching also reflects this. Numeracy is taught is functional way and has a definite focus on applying and making numeracy as functional as possible. For some pupils, especially those operating previously in the formal curriculum qualifications will provide a definite pathway into future college and work opportunities. For pupils accessing formal qualifications learning how to complete course work or an exam will need to be taught. An onus on community learning will be prevalent as this will enable young people to be as independent as possible when they leave, it should be noted that classroom learning will continue to provide the bedrock for this application to be successful

Vocabulary

At Kiora Hall, we understand the vital role that consistent mathematical language plays in supporting the learning journey of our learners. Mathematical language is more than just words; it is a critical tool that bridges understanding and empowers our pupils to engage confidently with mathematical concepts. Our commitment to employing consistent mathematical language is guided by the following principles:

- Providing Clear and Predictable Structure
- Enhancing Communication and Comprehension
- Supporting Conceptual Connections
- Reducing Anxiety and Cognitive Overload
- Promoting Inclusivity and Accessibility

At Kiora Hall, we are dedicated to creating a supportive and inclusive learning environment where mathematical language is used consistently to empower our learners. By doing so, we aim to enhance their learning experience and foster their growth as confident and capable individuals.

At Kiora Hall we have a number of pupils that have difficulties with communication. To support these learners, vocabulary will always be used at a stage appropriate level and communication aids (PECs, Objects of Reference and Symbols) will be used to support the learners' understanding.
Kiora Hall has a recommended maths vocabulary booklet as an aid for all staff.

Stage 0

Developing a strong grounding in number is essential so that all learners develop the necessary building blocks to excel mathematically. Learners should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers (Statutory Framework 2021)

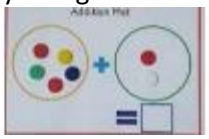
Addition	Subtraction
Learners will start their number journey through **subitising** – recognising quantities up to five without counting. Show quantities in familiar patterns (dice) and random patterns (scattered leaves). Ask the key question: what do you see? Learners are encouraged to gain a sense of the number system through the use of counting concrete objects and using fives and tens frames. Learners will also learn to match numbers to objects.	Learners will start exploring subtraction through counting backwards in 1's, 2's and 5s and 10's using stories, rhymes and songs (5 little monkeys, 10 green bottles). Learners will then learn to physically remove 'one' or more using objects and fives frames.





Learners need to be able to confidently combine objects in practical ways and count how many 'altogether'.





Learners should understand addition as counting on. They will count on in ones and twos using objects, cubes, bead strings, counting sticks and number lines. It is also important that learners are able to count on their fingers in a consistent way.



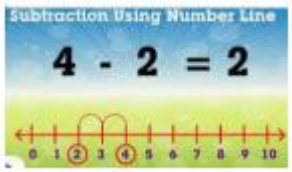


EYFS learners will begin to use + and = . They are encouraged to develop a mental picture of the number system in their heads to use for calculations. Higher attaining learners may be able to represent their calculations using symbols and numbers within a written calculation.



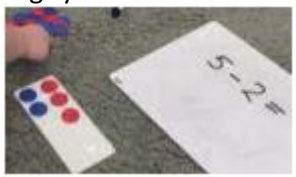



Learners will begin to count back in ones and twos using objects, cubes, bead strings, counting sticks and number lines. Learners should have the chance to record using marks they can explain and apply meaning.





EYFS learners will use concrete and pictorial representation to record their calculations. They begin to use - and = . Higher attaining learners may be able to represent their calculations using symbols and numbers within a written calculation.



Stage 1

Addition

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line.*

To understand the + and = signs and missing numbers:

Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer' e.g.

2 = 1 + 1 2 + 3 = 4 + 1

3 = 3 2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places e.g.

Subtraction

Key Vocabulary: *equal to, take away, less, minus, subtract, leave, difference between, how many more, fewer/less than, most, least, count back, count on, how many left, how much less.*

To understand the + and = signs and missing numbers: e.g.

7 - 3 = □ □ = 7 - 3

7 - □ = 4 4 = □ - 3

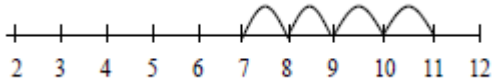
Use of pictures, marks and concrete objects:

Learners understand subtraction as **'take away'**: Sam spent 4p. What was his change from 10p?

3 + 4 = □ □ = 3 + 4
3 + □ = 7 7 = □ + 4
□ + 4 = 7 7 = 3 + □

Use prepared number lines and manipulatives:

7 + 4 = 11



Learners should be encouraged to make their own jumps using rulers, fingers, pens, etc.

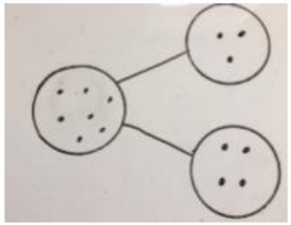
Bead strings and bead bars can be used to illustrate addition including through ten by counting on 2 then counting on 3:

8 + 5



Pictorial:

Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.



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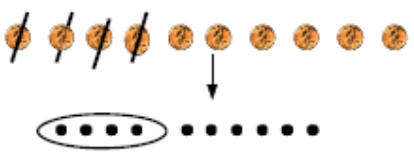
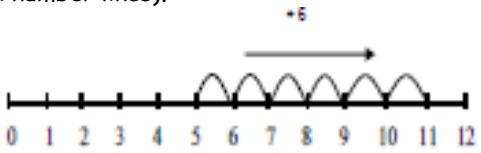
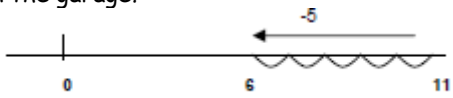


Find a '**difference**' by counting up: I have saved 5p. The socks I want to buy cost 11p. How much more do I need to buy the socks? (Learners are encouraged to draw jumps on prepared number lines).

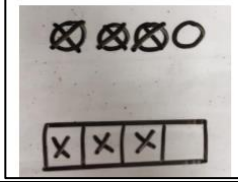


Use practical and informal written methods to support the subtraction of a one- digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number e.g. I have 11 toy cars. I have 5 cars too many to fit in the garage. How many cars fit in the garage?



Pictorial:

Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



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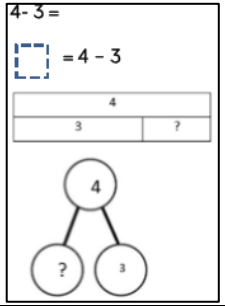


Stage 2

Addition

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens column.*

To understand the + and = signs and missing numbers:

As in Year 1 but with appropriate larger numbers e.g.

14 + 5 = 10 + □ **and** 32 + □ + □ = 100 **and** 35 = 1 + □ + 5

Subtraction

Key Vocabulary: *equal to, take, take away, less, minus, subtract, leave, difference between, how many more, fewer/less than, most, least, count back, count on, how many left, how much less?, strategy, partition, tens, ones.*

To understand the + and = signs and missing numbers:

Continue to use a range of equations as in Year 1 but with appropriate numbers for Year 2. Extend to numbers such as 14 + 5 = 20 - □

Find a small difference by counting on:

e.g. 42 – 39 = 3

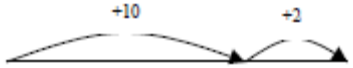
Partitioning:

Learners also need to partition into tens and ones and recombine e.g.

12 + 23 = 10 + 2 + 20 + 3
= 30 + 5
= 35

Learners should be taught how to count on in tens and ones, using a number line e.g.

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35



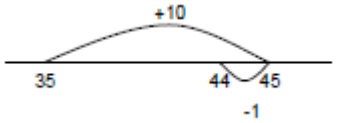
Learners should also be given additions that bridge through multiples of ten e.g.
Learners should be able to partition the 7 to relate adding 2 first and then 5

8 + 7 = 15

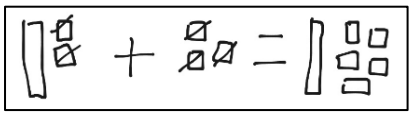


Adjusting:

Add 9 or 11 by adding 10 and adjusting by 1 e.g. 35 + 9 = 44



Pictorial:



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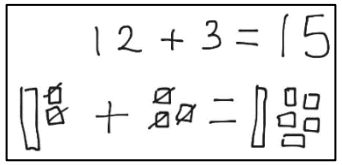
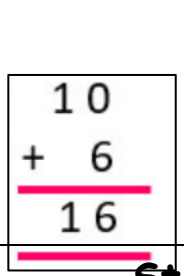
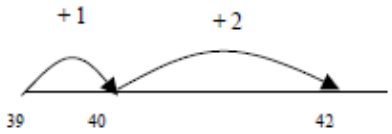


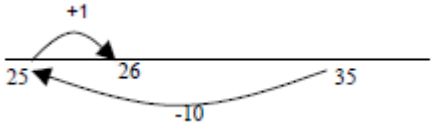
10
+ 6
16





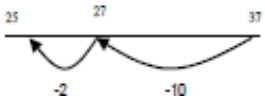
Adjusting:

Learners subtract 9 and 11 by adding 10 and adjusting. Begin to add 19/21 e.g: 35 – 29 = 3



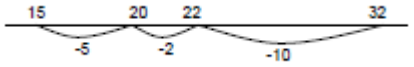
Use known number facts and place value to subtract: (Partition second number only)

37 – 12 = 37 – 10 – 2
= 27 – 2
= 25

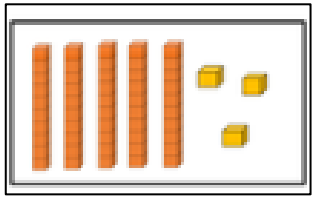


Bridge through ten where necessary:

32—17 = 15



Pictorial:



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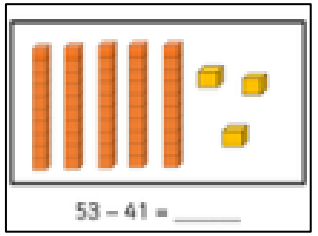


28
– 5
23

Stage 3

Addition

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary.*

To understand the + and = signs and missing numbers:

Learners need to continue using a range of equations as suggested in Year 1 and 2 but with appropriate larger numbers.

Partitioning:

Subtraction

Key Vocabulary: *equal to, take, take away, less, minus, subtract, leave, difference between, how many more, fewer/less than, most, least, count back, count on, how many left, how much less?, strategy, partition, tens, ones.*

To understand the + and = signs and missing numbers:

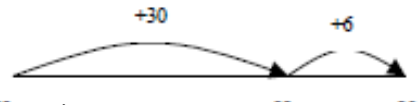
Continue to use a range of equations as in Year 2 but with appropriate numbers for Year 3.

Find a small difference by counting on:

e.g. 102 – 97 = 5

Learners also need to partition into tens and ones and recombine the numbers (second number).

53 + 36 = 53 + 30 + 6
= 83 + 6
= 89



Add a near multiple of ten to a 2-digit number:
Secure mental methods by using a number line to model the method. Continue as in Year 2 but with appropriate numbers e.g. 235 + 19 is the same as 235 + 20 – 1.

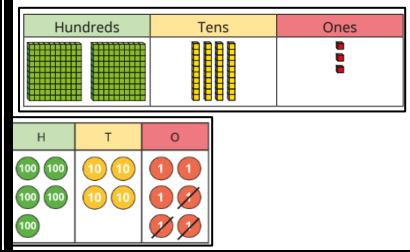
Formal methods of columnar addition to add numbers with 2 and 3-digits:
Partitioning to add – this should lead into a more contracted column method. It is important to add the ones before the tens to lead into the column method e.g.

Horizontal expansion:

83 + 42 = 125
80 + 3
+ 40 + 2
120 + 5 = 125

To support, pupils may physically carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Pictorial:



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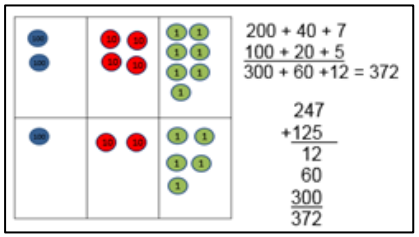
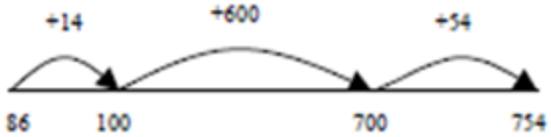


Subtract a near multiple of 10 to or from a two and three-digit number:
178 – 49 is the same as 178 – 50 + 1

Complimentary addition:
754 – 86 = 668



Formal methods of columnar subtraction to subtract numbers with 2 and 3-digits:
Partitioning to subtract – this should lead into a more contracted column method. It is important to subtract the ones before the tens to lead into the column method. Examples where there is no 'exchange' should be used first e.g. 89 – 35 = 54

80 and 9 (numbers should be written as 80 **and** 9, not 80 + 9).
- 30 and 5
50 and 4

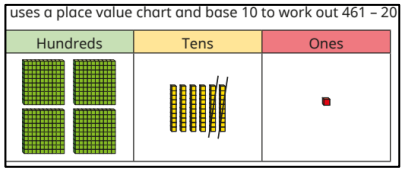
When learners are secure with this, 'exchanging' can take place, using manipulatives to aid understanding:



70 and 2 subtract 40 and 7:

60 and 12
~~70~~ and 2
-40 and 7
20 and

Pictorial:



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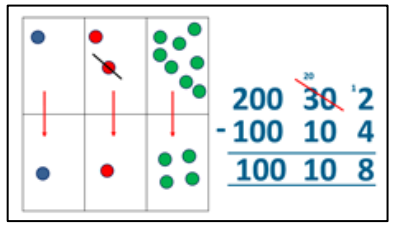
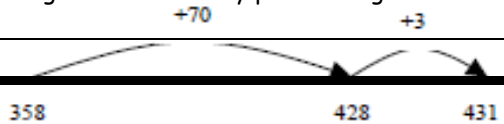


Stage 4

Addition

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, thousands, hundreds, digits, inverse, decimal point, decimal place, tenths.*

Partitioning:
Learners also need to partition into thousands, hundreds, tens and ones and recombine the numbers. Learners should also be taught to count on by partitioning the second number only. e.g.



Subtraction

Key Vocabulary: *equal to, take, take away, less, minus, subtract, leave, difference between, how many more, fewer/less than, most, least, count back, count on, how many left, how much less?, strategy, partition, tens, ones.*

Finding the difference by counting on:
This can be modelled on an empty number line e.g. 806 – 799 = 7

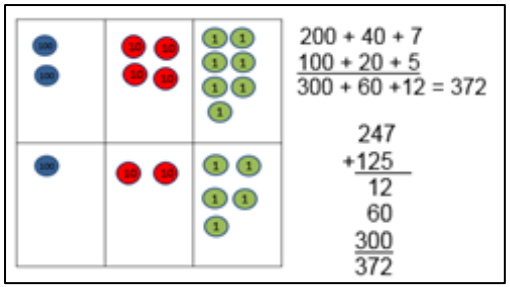
Subtract a near multiple of 10 and adjust:

358 + 73 = 358 + 70 + 3
428 + 3
= 431

Add a near multiple of 10 or 100 and adjust:
Continue as in previous stages but with appropriate numbers
e.g. 1458 + 79 is the same as 1458 + 80 - 1

Formal methods of columnar addition to add numbers with at least 4 digits:
Learners will move from the expanded method to the compact method for addition, adding **ones** first and then carrying numbers **underneath** the calculation. Learners will also add money and measures in context. Learners need to ensure that deciamals and numbers are lined up accurately and the decimal point goes in its own square. Learners will use mulnipulatives – dienes apparatus and place value counters. Teachers will model the compact method against the expanded method so that learners can see the similarities between the two methods. If some learners are not ready, continue with the methods established in Year 3.

Pictorial:



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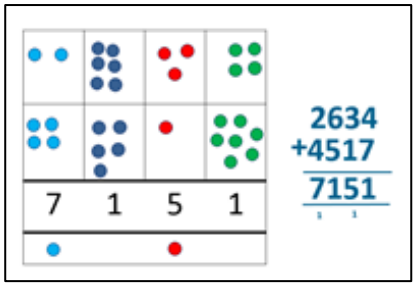
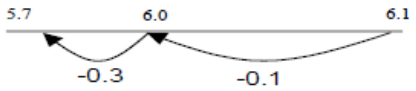
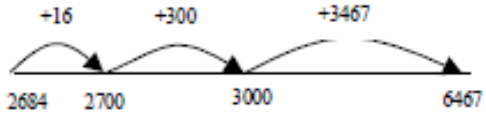


Continue as in previous year groups but with numbers appropriate to Year 4.

Use known facts and place value to subtract:
e.g. 6.1 – 0.4 = 5.7

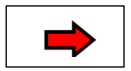


Complimentary addition:
e.g. 6467 – 2684 = 3783



Formal methods of columnar subtraction to subtract numbers with 4-digits:
Learners should subtract numbers with at least 4-digits. This should be extended to decimal numbers. The expanded method from Year 3 will be used with learners who are experiencing difficulties with decomposition and need to 'see' what is happening to each of the numbers e.g. 2754 – 1562 = 1192

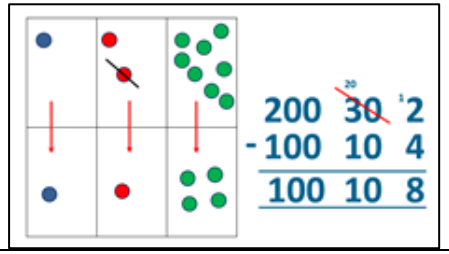
2000 and 700 and 150 and 4
-1000 and 500 and 60 and 2
1000 and 100 and 90 and 2



27 15 4
-1 5 6 2
1 1 9 2

Once again it is important to emphasise that the value of 2754 hasn't changed but that it has been partitioned in a different way e.g. 2000 and 600 and 150 and 4.

Pictorial:



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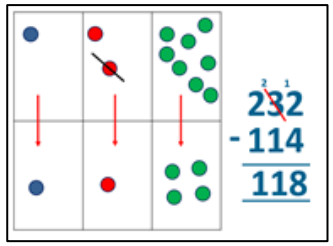


Stage 5 & 6

Addition

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary, thousands, hundreds, digits, inverse, decimal point, decimal place, tenths, hundredths, thousandths.*

Partitioning:

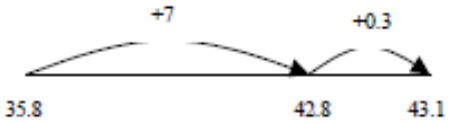
Subtraction

Key Vocabulary: *equal to, take, take away, less, minus, subtract, leave, difference between, how many more, fewer/less than, most, least, count back, count on, how many left, how much less?, strategy, partition, tens, ones, inverse.*

Finding the difference by counting on:
This can be modelled on an empty number line e.g. 8000 – 2785 = 5215
To make this method more efficient, the number of steps should be reduced.

To aid mental calculations, learners should be able to partition numbers up to and including 1,000,000 by the end of Year 6, and decimal fractions and recombine.
Learners either partition both numbers or just the second number and recombine e.g.

35.8 + 7.3 = 35.8 + 7 + 0.3
= 42.8 + 0.3
= 43.1



Round to a near multiple and adjust:
Continue as in previous stages but with appropriate numbers. Look at decimal numbers, adding and adjusting e.g. 0.99, 3.9, 4.01, etc.

Formal written methods of columnar addition:
Learners will continue with the compacted method of addition, adding numbers that exceed 4- digits. When dealing with decimal numbers, learners should be taught how to ensure that decimal points and numbers are aligned accurately.

789 + 642 becomes

```
  7 8 9
+ 6 4 2
-------
1 4 3 1
  1 1
```

Answer: 1431

Place value counters can be used, as in Year 4, alongside the columnar method to develop understanding of addition with decimal numbers.

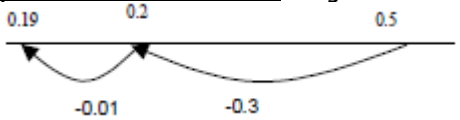
Teachers will use the terms, 'tenths, hundredths and thousandths' when adding decimals:

```
  2 3 . 3 6 1
    9 . 0 8
  5 9 . 7 7
+   1 . 3
-----------
  9 3 . 5 1 1
  2 1   2
```

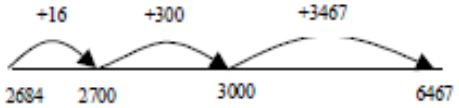
In Year 6, learners will also continue to look at missing number/digit problems: e.g. □ and # each stand for a different number. # = 34. # + # = □ + □ + #.
What is the value of □? What if # = 28? What if # = 2.
10 000 000 = 9 000 100 + □, 7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15

Subtract a near multiple of 10, 100 or 1000 and adjust:
Continue as in previous year groups but with numbers appropriate to Year 5 and 6.

Use known number facts and place value to subtract: e.g. 0.5 – 0.31 = 0.16



Complimentary addition:
6467 – 2684 = 3783



Formal methods of columnar subtraction to subtract numbers with more than 4-digits and decimals:
Learners will continue with the decomposition method of subtraction, subtracting numbers that exceed 4- digits. When dealing with decimal numbers, learners should be taught how to ensure the decimal points and numbers are aligned accurately. Continue to use place value counters to support.

874 – 523 becomes

```
  8 7 4
– 5 2 3
-------
  3 5 1
```

Answer: 351

932 – 457 becomes

```
  8  12  1
  9  3  2
– 4  5  7
---------
  4  7  5
```

Answer: 475

Stage 0

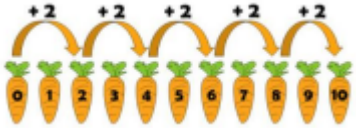
Multiplication	Division
Learners should begin to learn multiplication by using concrete objects to make and count equal groups of objects. Learners will recognise when given a set of equal amounts (we both have two counters each). Learners will also make links to pairs in songs and stories (Cinderella has two slippers).	Learners will start their journey by sharing and matching 1 to 1. For example, 5 cups for 5 people, 10 pieces of fruit for 10 learners, matching place settings for a teddy bear's picnic (knife, fork, plate).



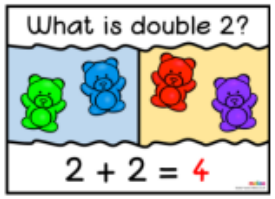
Learners need to be able to confidently match equal groups of varying objects.



Learners will count on in twos using a bead string and number lines.



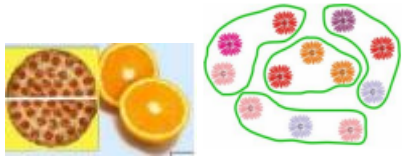
Learners will be able to double quantities (double one cube, double two bears). Then progress to doubling as repeated addition. 2 + 2 = 4.



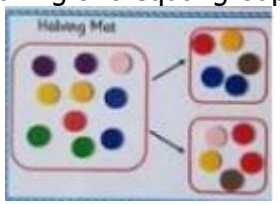
EYFS learners will have opportunities to use concrete and pictorial representation to record their calculations.



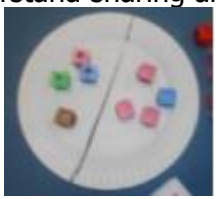
Progressing onto using concrete objects and share equally into two. Learners will also be able to group these objects into equal groups.



Learners will be able to count a set of objects up to 10 and halve them by making two equal groups.



Learners will need to understand sharing and halving as dividing by 2.



EYFS learners will have opportunities to use concrete and pictorial representation to record their calculations.



Stage 1

Multiplication	Division
Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, communatative.*	*Key Vocabulary:* *share, share equally, one each, two each…, group, groups of, lots of, array*

Multiplication is related to doubling and counting groups of the same size.
Use arrays to understand multiplication can be done in any order (commutative)



2 x 4 = 8



4 x 2 = 8



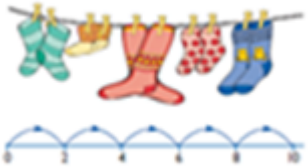
4 x 2 = 8

2 x 4 = 8

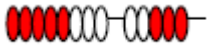
Looking at columns and looking at rows 2 + 2 + 2, 3 + 3
3 groups of 2 and 2 groups of 3

Counting using a variety of practical resources:
Counting in 2s e.g. counting socks, shoes, animal's legs…
Counting in 5s e.g. counting fingers, fingers in gloves, toes…
Counting in 10s e.g. fingers, toes…

2 + 2 + 2 + 2 + 2 = 10
2 x 5 = 10
2 multiplied by 5
5 pairs
2 groups of 5
5 hops of 2

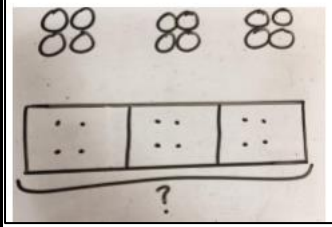


Counting steps using bead strings and on prepared number lines.



Pictorial:

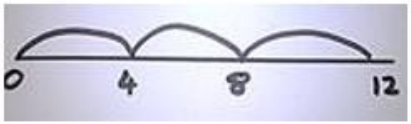
Children to represent the practical resources in a picture and use a bar model.



Abstract:

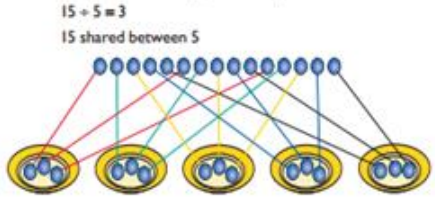
Abstract number line showing three jumps of four.

3 × 4 = 12



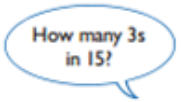
Learners must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Learners should be given opportunities to reason about what they notice in number patterns.

Group AND **share small quantities- understanding the difference between the two concepts.** Learners to continue to use objects, pictures and marks to record.



Sharing

Develops importance of one-to-one correspondence. Learners should be taught to share using concrete apparatus.

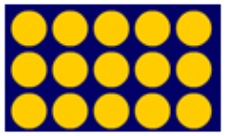


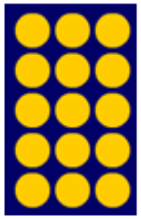


15 ÷ 3 = 5

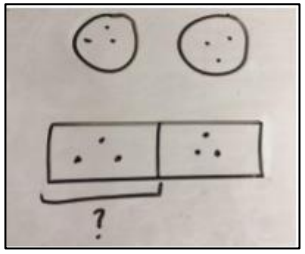
Grouping

Learners should apply their counting skills to develop some understanding of grouping.
Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.





Pictorial:



Abstract:

6 ÷ 2 = 3

3	3

Children should also be encouraged to use their 2 times tables facts.

Stage 2

Multiplication

Division

Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition.*

x = signs and missing numbers:

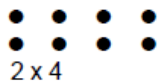
7 x 2 = □ □ = 2 x 7

7 x □ = 14 14 = □ x 7

□ x 2 = 14 14 = 2 x □

□ x △ = 14 14 = □ x △

Develop understanding of multiplication using array and number lines. Include multiplications not in the 2, 5 or 10 times tables.

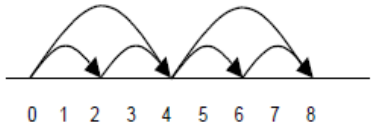


4 x 2 or 4 + 4

2 x 4

Or repeated addition

2 + 2 + 2 + 2



Begin to develop understanding of multiplication as scaling (3 times bigger/taller)





Doubling numbers up to 10 + 10. Link with understanding scaling and using known doubles to work out double 2d numbers e.g. (double 15 = double 10 + double 5)

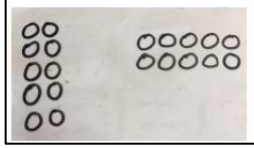
Work towards written methods using partitioning:

Learners need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so e.g. Double 6 is the same as double five add double one.

Use jottings to develop an understanding of doubling two digit numbers

Pictorial:

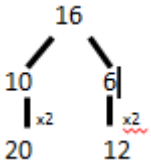
Children to represent the arrays pictorially.



Abstract:

Children to be able to use an array to write a range of calculations e.g.

10 = 2 × 5
5 × 2 = 10
2 + 2 + 2 + 2 + 2 = 10
10 = 5 + 5



Key Vocabulary: *share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over.*

÷ = signs and missing numbers

6 ÷ 2 = ϒ ϒ = 6 ÷ 2

6 ÷ ϒ = 3 3 = 6 ÷ ϒ

ϒ ÷ 2 = 3 3 = ϒ ÷ 2

ϒ ÷ ∇ = 3 3 = ϒ ÷ ∇

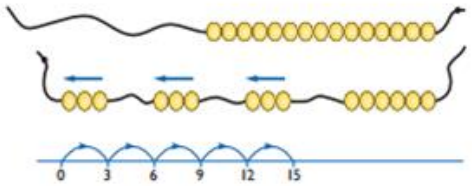
Know and understand sharing and grouping- introducing learners to the ÷ sign.

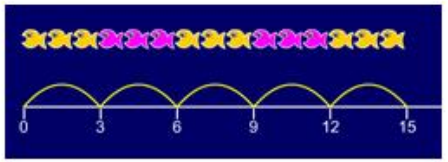
Learners should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Sharing and grouping using a numberline

Group from zero in jumps of the divisor to find out 'how many groups of 3 are there in 15?'

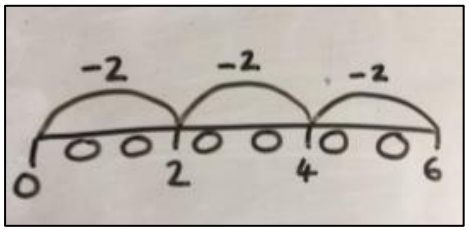
15 ÷ 3 = 5





Continue work on arrays. Support learners to understand how multiplication and division are inverse. Look at an array – what do you see?

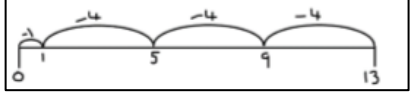
Pictorial:



Abstract:

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'



Stage 3

Multiplication

Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, ones, value.*

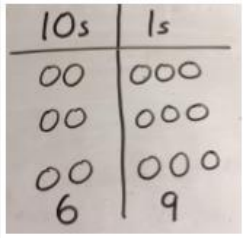
x = signs and missing numbers:
Continue using a range of equations as in Year 2 but with appropriate numbers.
Know by heart x2, x3, x4, x5, x8 and x10.
Multiply a single digit by 1, 10, 100

Arrays and repeated addition:
Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).
Derive related facts
7 x 5 = 35
5 x 7 = 35
35 ÷ 7 = 5
35 ÷ 5 = 7

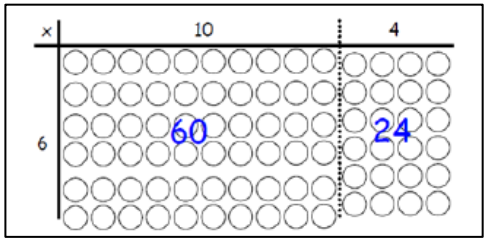
Written methods (progressing up to 2d x 1d)
Developing written methods using understanding of visual images.
Develop onto the grid method:

Pictorial:
(Place value counters for 3x23)

Children to represent the counters pictorially.



Abstract:
(Grid method for 6x14)



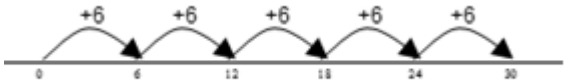
Give learners opportunities for learners to explore this and deepen understanding using Dienes apparatus and place value counters.

Division

Key Vocabulary: *share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over,* ***inverse, short division, remainder, multiple.***

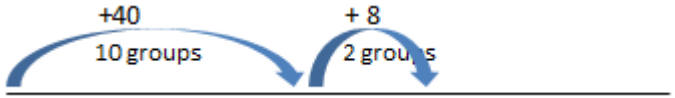
÷ = signs and missing numbers
Continue using a range of equations as in Year 2 but with appropriate numbers.

Understand division as sharing and grouping
30 ÷ 6 can be modelled as:

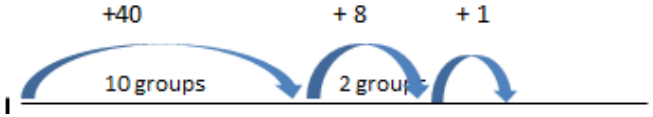


Sharing – 18 shared between 3 **OR** Grouping - How many 3's make 18?

Using a numberline:
Learners need to be able to partition the dividend in different ways.
48 ÷ **4 = 12**



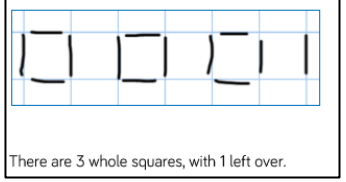
Remainders
49 ÷ 4 = 12 r1



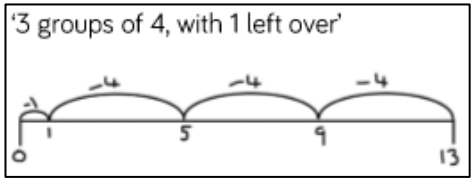
Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?
Place value counters can be used to support learners apply their knowledge of grouping.
E.g. 60 ÷ 10 = How many groups of 10 in 60? 600 ÷ 100 = How many groups of 100 in 600?

Pictorial:

Children to represent the lollipop sticks pictorially.


There are 3 whole squares, with 1 left over.

Abstract:



Stage 4

Multiplication

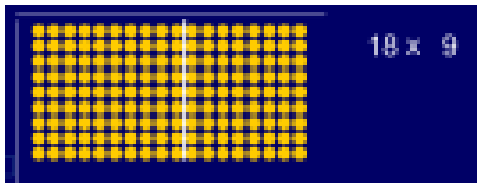
Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, ones, value, inverse.*

x = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.
Know by heart multiplications to 12 x 12.

Written methods and using partitioning

Continue to use arrays:

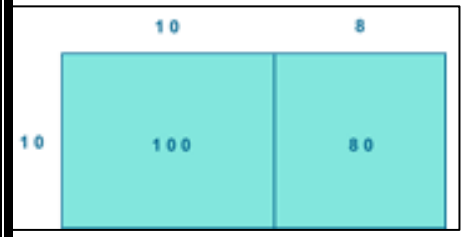


18 x 9 = 162
18 x 9 = (10 x 9) + (8 x 9) = 162

AND

Use the grid method of multiplication. Learners need to embed and deepen their understanding of the grid method to multiply up to **4d x 1d**. Ensure that this is still linked back to their understanding of arrays and place value counters.

Pictorial:



Abstract:

342 × 7 becomes

	3	4	2
×			7
2	3	9	4
	2	1	

Answer: 2394

Division

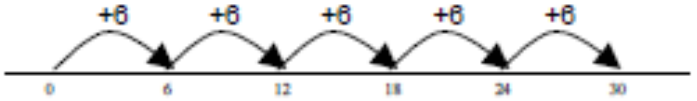
Key Vocabulary: *share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over,* ***inverse, short division, remainder, multiple.***

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 and 3 but with appropriate numbers.

Sharing and grouping

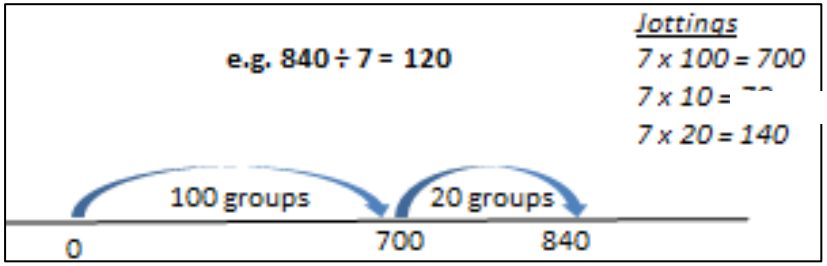
30 ÷ 6 can be modelled as:
grouping – groups of 6 placed on a number line and the number of groups counted e.g.



Written methods and remainders:

Learners should continue with this until they have a secure understanding. Learners should then progress in their written calculation methods by using known facts with which they are fluent and use a logical progression in the numbers they use. These should include calculations **with and without remainders**. Remainders should be interpreted according to the context i.e. rounded up or down to relate to the answer of the problem.

Pictorial:



Abstract (up to 3 digits):

432 ÷ 5 becomes

		8	6 r 2
5	4	3	2
		3	

Answer: 86 remainder 2

Stage 5

Multiplication

Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, ones, value, inverse.*

x = signs and missing numbers:
Continue using a range of equations but with appropriate numbers.

Mental methods
X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning).

Solving practical problems where learners need to scale up. Relate to known number facts.

Identify factor pairs for numbers.

Written methods and using partitioning progressing to 4d x 2d
47 x 6 = 282

47 x 6 = (40 x 6) + (7 x 6) = 282

Grid method as in Year 4 using numbers appropriate for Year 5 and place value counters, leading to short multiplication then long multiplication:

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

342 × 7 becomes

$$\begin{array}{r} 3\ 4\ 2 \\ \times\quad\quad 7 \\ \hline 2\ 3\ 9\ 4 \\ \hline {\scriptstyle 2\ \ 1} \end{array}$$

Answer: 2394

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2}\ \ \\ 2\ 4 \\ \times\ 1\ 6 \\ \hline 2\ 4\ 0 \\ 1\ 4\ 4 \\ \hline 3\ 8\ 4 \end{array}$$

Answer: 384

Division

Key Vocabulary: *share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, remainder, multiple, quotient, prime number, prime factors, composite number (non-prime).*

÷ = signs and missing numbers:
Continue using a range of equations as in Year 2 and 3 but with appropriate numbers.

Sharing and grouping:

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders:
Learners begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows learners to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Written methods (up to 4 digits):
Learners need to continue to use the chunking method for division as shown using the number line and jottings in Year 4. This will then develop into short division and long division as and when the learners are secure in their understanding:

432 ÷ 5 becomes

$$\begin{array}{r} 8\ \ 6\ \text{r}\ 2 \\ 5\overline{)\ 4\ 3\ {\scriptstyle 3}\ 2} \end{array}$$

Answer: 86 remainder 2

432 ÷ 15 becomes

$$\begin{array}{r} \quad\ 2\ 8 \\ 1\ 5\overline{)\ 4\ 3\ 2} \\ \quad\ 3\ 0\ 0 \quad 15\times20 \\ \hline \quad\ 1\ 3\ 2 \\ \quad\ 1\ 2\ 0 \quad 15\times8 \\ \hline \quad\ 1\ 2 \end{array}$$

Stage 6

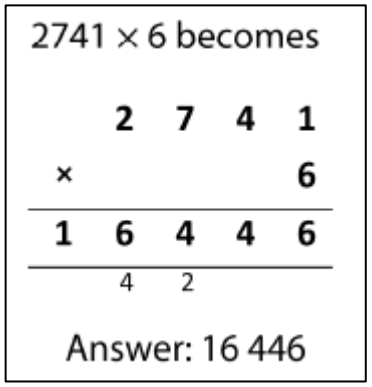
Multiplication

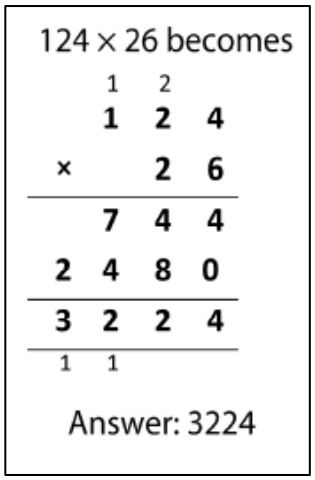
Key Vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, ones, value, inverse, tenths, hundredths, decimal.

x = signs and missing numbers:
Continue using a range of equations but with appropriate numbers.

Mental methods;
Identifying common factors and multiples of given numbers
Solving practical problems where learners need to scale up. Relate to known number facts.

Written methods:
Continue to refine and deepen understanding of written methods including fluency for using long multiplication, continuing to link this with grid multiplication as in Year 5;





Division

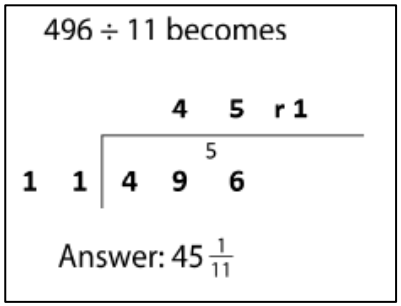
Key Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, remainder, multiple, quotient, prime number, prime factors, composite number (non-prime), common factor.

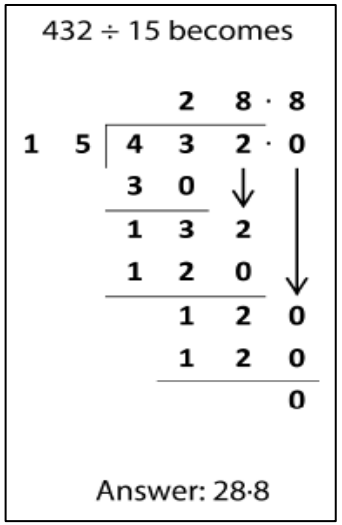
÷ = signs and missing numbers:
Continue using a range of equations as in Year 2 and 3 but with appropriate numbers.

Sharing and grouping:
Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders:
Quotients expressed as fractions or decimal fractions
61 ÷ 4 = 15 ¼ or 15.25
Learners begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows learners to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Written methods:
Learners to develop greater fluency in short and long division methods:





Fractions

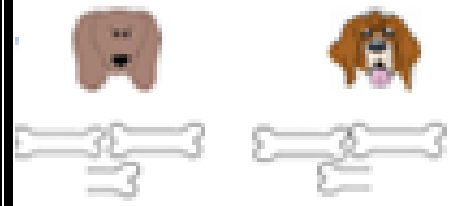
Stage 0

Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds.

Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young learners to fractions and calculating with them.

Setting problems in real life contexts and solving them with concrete apparatus will support learners with their understanding.

'I have got 5 bones to share between my two dogs. How much will each dog get?'



Stage 1

Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters.

To recognise, find and name a half as one of two equal parts of an object, shape or quantity:

Learners will use objects to find halves of different amounts as well as halves of shapes, understanding that both sides have to be equal e.g.

How many in half the set of pencils?



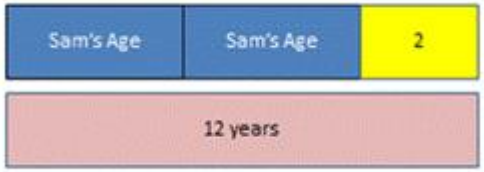
Learners will begin to shade in halves and quarters of simple shapes.

Stage 2

Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters,

To recognise, find, name and write fractions $^{1}/_{3}$, $^{1}/_{4}$, $^{2}/_{4}$ and $^{3}/_{4}$ of a length, shape, set of objects or quantity

Learners will continue with using manipulatives to aid their understanding of simple fractions. Learners to use the bar method to help solve problems:

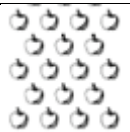


To write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $^{2}/_{4}$ and $\frac{1}{2}$

Learners can have a go at recording the calculation they have carried out: 2 ½ + 2 ½ = 5	Learners will also be taught how to find a quarter of different amounts of objects by physically dividing and sharing different amounts.	e.g. Would a person prefer 2/4 or ½ of a pizza? Explain your answer.

Fractions

Stage 3	Stage 4	Stage 5	Stage 6
Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters, fifths, sixths, tenths.	Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters, fifths, sixths, tenths, hundredths, equivalent.	Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters, fifths, sixths, tenths, hundredths, equivalent.	Key Vocabulary: Share, share equally, in ones, in twos, in threes, half, halves, third, thirds, quarters, fifths, sixths, tenths, hundredths, equivalent.
To count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10	To recognise and show, using diagrams, families of common equivalent fractions.	To compare and order fractions whose denominators are all multiples of the same number and recognise mixed and improper fractions, converting one to another. They will also need to multiply proper fractions and mixed numbers by whole numbers.	To be able to use common factors to simplify fractions; use common multiples to express fractions in the same denomination
Learners will be finding and writing fractions of sets of objects and write fractions of shapes.  Learners will also be finding fractions of different amounts and place fractions on a number line. Here are 18 apples. Put a ring around 1/3 of them.	e.g. To recognise that ½ is the same as 5/10.  Learners will be dividing numbers by 10 and 100 to generate tenths and hundredths of a number. They will also be calculating increasingly harder fractions to find answers with whole amounts.	e.g. What is $^{3}/_{10}$ of: 50, 20, 100…?	Learners will be comparing fractions and associating fractions with division in order to calculate decimal fraction equivalents: What fraction of 1 litre is 413 ml? They will also be adding and subtracting fractions with different denominations and mixed numbers using the concept of equivalent fractions.

	$\frac{1}{4}$ of 24 = $\frac{1}{2}$ of $\square$		They will also be multiplying simple pairs of proper fractions, writing its answer in the simplest form.